Adiabatic three-wave volume hologram: large efficiency independent of grating strength and polarization

C. C. Tsai, L. B. Glebov, and B. Ya. Zeldovich

College of Optics & Photonics / CREOL, University of Central Florida, 4000 Central Florida Boulevard, Orlando, Florida 32816-2700

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A transmission hologram with two volume gratings is considered in the regime of wave A diffracted into wave B via an intermediate weakly excited wave C. In analogy to the phenomenon of stimulated Raman adiabatic passage in nonlinear optics, this new scheme demonstrates diffraction efficiency with low sensitivity to the hologram's strength. A theory of three-wave adiabatic coupling has been developed and explored analytically. Numerical results show an example of a coupling profile that preserves high diffraction efficiency with almost no dependence on the hologram's strength, including the suppressed influence of polarization. © 2006 Optical Society of America

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Transmission volume holograms have a variety of applications, such as optical switching, filtering, multiplexing, imaging systems, and spectral beam combining. The diffraction efficiency of a single thick transmission grating is $\eta=[\sin(M)]^2$,[1–3] which can reach 100% if hologram strength $M=\kappa L\equiv\pi n_1 L/\lambda_{\mathrm{vac}}$ is $\pi/2=1.57, 3\pi/2, 5\pi/2, \ldots$. Here L is the effective interaction length, $L=(\mathbf{e}_A\cdot\mathbf{e}_B)L_z/[\cos(\theta_{\mathrm{A,med}})\cos(\theta_{\mathrm{B,med}})]^{0.5}$, $(\mathbf{e}_A\cdot\mathbf{e}_B)$ is the polarization factor, and the modulation of the refractive index in the volume grating is assumed to be $\delta n=n_1\cos(\mathbf{Q}\cdot\mathbf{R})$; λ_{vac} is the wavelength of light in vacuum, and $\theta_{A,B,\mathrm{med}}$ are the angles (inside the medium) of the $\mathbf{k}$ vectors of waves A and B with the z axis. A new material for volume holographic gratings, namely, photothermorefractive glass, was recently developed at the College of Optics and Photonics/Center for Research and Education in Optics and Lasers.[4–6] This material has already yielded almost 100% diffraction efficiency for both transmission and reflection holograms.

A disadvantage of a simple transmission volume hologram is the rather high sensitivity of the diffraction efficiency η to hologram strength $M=\kappa L$. This sensitivity may cause certain problems in the process of manufacturing the hologram. In this Letter we propose a new scheme based on an analogy with the nonlinear optical phenomenon STIRAP, stimulated Raman adiabatic passage.[7]

In applying the parallel thinking of the stimulated Raman adiabatic passage to holography, we introduce a third intermediate wave, C, into consideration, which interacts both with primary wave A and diffracted wave B to accomplish energy transfer from A to B. Therefore two gratings are needed if we are to have three coupled waves in our new volume hologram. The two gratings are characterized by the following modulation of index of refraction:

$$\begin{aligned}\delta n &= n_{CA}(z)\cos(\mathbf{Q}_{CA}\cdot\mathbf{R})+n_{CB}(z)\cos(\mathbf{Q}_{CB}\cdot\mathbf{R})\\ &=0.5[n_{CA}(z)\exp(i\mathbf{Q}_{CA}\cdot\mathbf{R})+n_{CB}(z)\exp(i\mathbf{Q}_{CB}\cdot\mathbf{R})]\\ &\quad+\text{complex conjugate}. \qquad (1)\end{aligned}$$

Here C is the intermediate wave, as shown in Fig. 1. We present our field (for definiteness, of the TE polarization) in the form

$$\mathbf{E}_{\mathrm{real}}(\mathbf{R},t)=\mathbf{E}(\mathbf{R},t)+[\mathbf{E}(\mathbf{R},t)];$$

$$\begin{aligned}\mathbf{E}(\mathbf{R},t)=\frac{\hat{\mathbf{e}}_y}{2}\Bigg\{&\frac{a(z)}{[\cos(\theta_{A,\mathrm{med}})]^{1/2}}\exp(i\mathbf{k}_A\cdot\mathbf{R})\\ &+\frac{b(z)}{[\cos(\theta_{B,\mathrm{med}})]^{1/2}}\exp(i\mathbf{k}_B\cdot\mathbf{R})\\ &+\frac{c(z)}{[\cos(\theta_{C,\mathrm{med}})]^{1/2}}\exp(i\mathbf{k}_C\cdot\mathbf{R})\Bigg\}\\ &\times\exp[i\,\delta k_x x-i(\omega+\delta\omega)t]. \qquad (2)\end{aligned}$$

Equation (2) was written on the assumption that

$$\mathbf{k}_{\mathrm{A,B,C}}=(\omega n_0/c)[\mathbf{e}_x\sin(\theta_{\mathrm{A,B,C,med}})+\mathbf{e}_z\cos(\theta_{\mathrm{A,B,C,med}})],$$

$$\theta_{\mathrm{B,med}}=-\theta_{\mathrm{A,med}},\quad \theta_{\mathrm{C,med}}=0,$$

$$\mathbf{Q}_{CA}=\mathbf{k}_C-\mathbf{k}_A,\quad \mathbf{Q}_{CB}=\mathbf{k}_C-\mathbf{k}_B, \qquad (3)$$

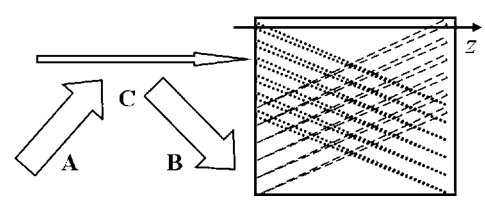

Fig. 1. Configuration of three-wave interaction inside a double-recorded volume hologram with modulation of coupling coefficients. The counterintuitive order in which these gratings are turned on and off should be emphasized: dotted lines, $\kappa_{CB}(z)$; dashed lines, $\kappa_{CA}(z)$.

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i.e., that for unperturbed propagation directions $\mathbf{k}_A$, $\mathbf{k}_B$, and $\mathbf{k}_C$ (inside the medium of the hologram) the Bragg condition is satisfied when the frequency detuning $\delta\omega$ and the angular detuning $\delta\theta_{A,\mathrm{air}}$ are zero. It should be noted also that the particular configuration is A/B symmetric, while the general nonsymmetric case yields similar results. Parameter δk_x is influenced by both the frequency detuning $\delta\omega$ and the change in angle of incidence $\delta\theta_{A,\mathrm{air}}$:

$$\delta k_x = \frac{\delta\omega}{c}\sin(\theta_{A,\mathrm{air}}) + \frac{\omega}{c}\cos(\theta_{A,\mathrm{air}})\delta\theta_{A,\mathrm{air}}. \quad (4)$$

The coupled-wave equations are

$$\begin{aligned} \mathrm{d}a/\mathrm{d}z &= i[\kappa_{CA}(z)]^* c(z) + i\alpha a(z), \\ \mathrm{d}b/\mathrm{d}z &= i[\kappa_{CB}(z)]^* c(z) + i\beta b(z), \\ \mathrm{d}c/\mathrm{d}z &= i\kappa_{CA}(z)a(z) + i\kappa_{CB}(z)b(z) + i\gamma c(z), \end{aligned} \quad (5)$$

Here

$$\alpha = \left[\frac{\omega}{c}(\xi-\mu)\frac{\delta\omega}{\omega} - v\,\delta\theta_{A,\mathrm{air}}\right], \quad \beta = \left[\frac{\omega}{c}(\xi+\mu)\frac{\delta\omega}{\omega} + v\,\delta\theta_{A,\mathrm{air}}\right], \quad \gamma = \frac{\omega}{c}\left(n\frac{\delta\omega}{\omega}\right), \quad \mu = \frac{\sin^2\theta_{A,\mathrm{air}}}{n\cos\theta_{A,\mathrm{med}}},$$

$$v = \frac{\sin\theta_{A,\mathrm{air}}\cos\theta_{A,\mathrm{air}}}{n\cos\theta_{A,\mathrm{med}}}, \quad \xi = \frac{\mathrm{n}}{\cos\theta_{A,\mathrm{med}}},$$

$$\kappa_{CA} = \frac{\omega}{2c[\cos(\theta_{A,\mathrm{med}})]^{1/2}} n_{CA},$$

$$\kappa_{CB} = \frac{\omega}{2c[\cos(\theta_{A,\mathrm{med}})]^{1/2}} n_{CB}. \quad (6)$$

The parameters characterizing the spectral and angular detuning from the Bragg regime are α, β, and γ, all of dimensions 1/m. $\theta_{A,\mathrm{air}} = |\theta_A| = |\theta_B|$ is the angle of incidence from air for both the A and the B waves, which are assumed to propagate symmetrically with respect to the z axis.

Analysis of the eigensolutions of Eq. (2) of the form $\exp(i\Lambda z)$ with constant values of κ_{AC} and κ_{BC} and under the condition of zero detuning ($\delta\omega/\omega = \delta\theta_{A,\mathrm{air}} = 0$, i.e., $\alpha = \beta = \gamma = 0$) yields three eigenvectors $[A, B, C]$:

$$[\kappa_{AC}^*/(|\kappa_{AC}|^2 + |\kappa_{BC}|^2)^{1/2}, \kappa_{BC}^*/(|\kappa_{AC}|^2 + |\kappa_{BC}|^2)^{1/2}, 1],$$

$$[-\kappa_{AC}^*/(|\kappa_{AC}|^2 + |\kappa_{BC}|^2)^{1/2}, -\kappa_{BC}^*/(|\kappa_{AC}|^2 + |\kappa_{BC}|^2)^{1/2}, 1],$$

$$[\kappa_{BC}, -\kappa_{AC}, 0] \equiv \mathrm{const.}[(1/\kappa_{AC}), -(1/\kappa_{BC}), 0]. \quad (7)$$

Most interesting is the third eigenmode, which has zero amplitude of the C wave. Remarkably, the A and B amplitudes in this mode have opposite signs and values inversely proportional to the corresponding coupling coefficients. Thus, the scattering processes of the A and B waves into C cancel each other, so that indeed $C = 0$ for this C dark mode.

The notion of eigenmodes is, strictly speaking, applicable to a system of equations with constant coefficients only. However, if these coefficients change slowly with z, then one can expect adiabatic evolution, i.e., that the amplitude of each mode (not of a wave but of a mode) will be preserved in the process of propagation.

Here we make three observations: First, the C dark mode has A and B waves only. Therefore the adiabatic evolution of this C dark mode means an energy exchange between A and B waves, with the small C amplitude as an intermediary, which arises only as a correction to the adiabatic approximation. Second, there is a possibility that a pure C dark mode will be excited by the pure input A wave if coefficient $\kappa_{CA}(z=0) = 0$ at input $z = 0$. Third, and most important, is that one should gradually increase $\kappa_{CA}(z)$ and diminish $\kappa_{CB}(z)$ to zero value at the output: $\kappa_{CB}(z = L) = 0$. The assumption that the changes are gradual should guarantee that the solution will still correspond to the C dark mode. Meanwhile, with $\kappa_{CB}(z = L) = 0$, all the energy will be transferred into a B wave at the output cross section $z = L$.

We stress that the order in which the gratings are to be turned on and off along the z coordinate is counterintuitive. It is the A wave that is present at the input; however, it is the other grating, $\kappa_{CB}(z=0) \neq 0$, that is turned on here. On the contrary, it is the B wave that we want to get at the output; however, it is the other grating, $\kappa_{CA}(z=L) \neq 0$, that is turned on there. To keep the validity of the notion of eigenmodes, we must change the strength of coupling coefficients slowly, or adiabatically. This actually requires that $\kappa(z)$ be a slowly varying function, i.e., one that satisfies the condition that $\mathrm{d}\kappa/\mathrm{d}z \ll |\kappa|^2$.

A potential advantage of this design is that, on the assumption of validity of the adiabatic approximation, the efficiency of $A \to B$ power transfer is 100% and does not depend on the hologram's strength M. In this case the values of M_{CA} and M_{CB} are defined as

$$M_{CA} = \int \kappa_{CA}(z)\mathrm{d}z, \quad M_{CB} = \int \kappa_{CB}(z)\mathrm{d}z. \quad (8)$$

We have validated our adiabatic reasoning by direct numeric integration of the coupled equations for all three waves, A, B, and C. From our numerical experiments, a preferred profile of $\kappa_{CB}(z)$ and $\kappa_{CA}(z)$ was found:

$$\kappa_{CB}(z) = M_{CB}[\cos(\pi z/2L_z)]^2(2/L_z),$$

$$\kappa_{CA}(z) = M_{CA}[\sin(\pi z/2L_z)]^2(2/L_z). \quad (9)$$

The dependence of diffraction efficiency $\eta(A \to B)$ on hologram strengths M_{CA} and M_{CB} in the intervals from 0 to 15 is depicted in Fig. 2 for the case of perfect Bragg matching. Our data show that diffraction efficiency η is greater than 85% when $(M_{CA}, M_{CB}) \geqslant (5.2, 5.2)$ and is greater than 97.85% when $(M_{CA}, M_{CB}) \geqslant (7.5, 7.5)$. Efficiency η asymptotically approaches 100% for larger hologram strengths. These results manifest the important feature of our

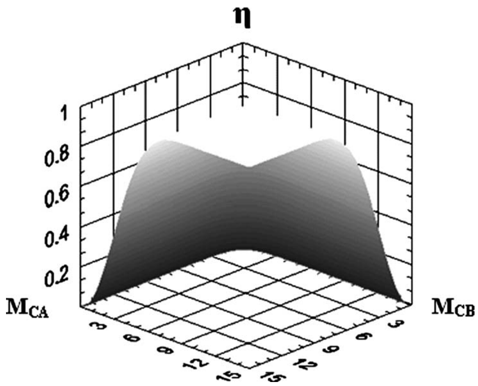

Fig. 2. Diffraction efficiency of the diffracted wave as a function of two hologram strengths, $\eta=\eta(M_{CA},M_{CB})$.

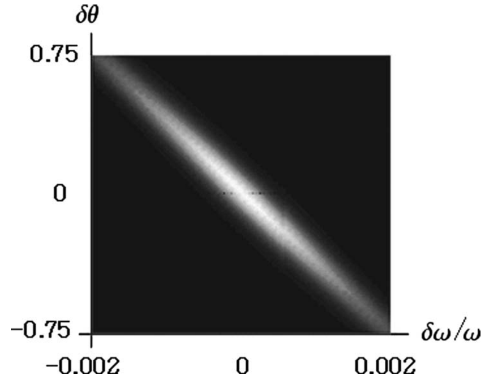

Fig. 3. Diffraction efficiency of the diffracted wave as a function of angular and spectral detuning values ($\delta\theta$, in degrees, in air; $\delta\omega/\omega$, dimensionless) with relatively small hologram strengths $M_{CA}=M_{CB}=4$, thickness $L=5$ mm, $\lambda_{\text{vac}}=1.06\ \mu$m, and $\theta_{A,\text{air}}=50°$. Maximum diffraction efficiency $\eta_{\max}$ of power transfer $A\rightarrow B$ is $\eta_{\max}=0.615$. There are both spectral and angular selectivity, as in a single-grating $A\rightarrow B$ hologram, but the compensation for wavelength detuning by angular detuning is no longer perfect.

scheme: dramatic reduction of the sensitivity of diffraction efficiency η to M. Other profiles have been studied for the same hologram strengths, M_{CA} and M_{CB}, with similar results; the profile from Eqs. (9) was the best.

Consideration of other (TM) polarization of interacting waves in our symmetric geometry results in multiplication of each of the coupling constants (at the given values of n_{CA} and n_{CB}) by the factor $(\mathbf{e}_A\cdot\mathbf{e}_C)\equiv(\mathbf{e}_B\cdot\mathbf{e}_C)=\cos(\theta_{A,\text{med}})$. This is quite remarkable since it allows TM-polarized wave A to be diffracted into wave B even when they are propagating at 90° to each other because $(\mathbf{e}_A\cdot\mathbf{e}_C)\equiv(\mathbf{e}_B\cdot\mathbf{e}_C)$ is 0.707 in each case. This is the task that could not be achieved with a single-grating hologram. Moreover, the intensity of the diffracted B wave will be the same, close to 100% (and even the phase will be the same), for both polarizations if the modified hologram strength is large enough for the unfavorable polarization as well.

In reality, perfect Bragg matching may not always be possible. We studied the effect of detuning on our new type of transmission hologram by numerical integration of the coupled-wave equations. We kept the same profile [Eqs. (9)] for the coupling but took detuning into account. The results of the numerical solution of Eqs. (5) are shown in Fig. 3. As the figure indicates, our adiabatic hologram possesses both spectral and angular selectivity, just as an ordinary volume hologram does. However, unlike for an ordinary hologram, one cannot substitute a change of wavelength for a change of angle without the loss of diffraction efficiency. The ratio of width $\delta\theta$ (at optimally tuned λ_{vac}) to width $\delta\theta$ (at fixed λ_{vac}) depends on the particular geometry and on the hologram strength. Our numerical experiments have shown that this factor is approximately 7–12. The same may be said for spectral selectivity.

The same ideas of adiabatic A/B interaction via a weakly excited intermediate C wave are applicable to the design of a beam splitter, $|A|^2\rightarrow p_A|A|^2+p_B|B|^2$, where the splitting ratio p_A/p_B is robust with respect to manufacturing errors and polarization.

In conclusion, we have suggested a new type of double-grating volume hologram that transfers energy from the primary wave into the diffracted wave through a weak intermediate wave. We consider the strongly diminished dependence of diffraction efficiency on the hologram's strength to be one of the advantages of the scheme suggested. Another possible advantage of the suggested scheme is its factor-of-2 smaller spatial frequency $|\mathbf{Q}_{\text{AC}}|$, $|\mathbf{Q}_{\text{BC}}|$ of each of the gratings if we have to achieve a certain value of the deflection angle, $\theta=2\arcsin(|\mathbf{k}_{\text{A}}-\mathbf{k}_{\text{B}}|/2k)$. Yet another possible advantage that the same optical element (our double-grating adiabatic hologram) can be used to handle diffraction of both polarizations efficiently.

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